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A novel biomechanical model assessing continuous orthodontic archwire activation

Christopher Canales,[a] Matthew Larson,[a] Dan Grauer,[b] Rose Sheats,[c] Clarke Stevens,[d] and Ching-Chang Ko[e]
Chapel Hill, NC, and Omaha, Neb

Introduction: The biomechanics of a continuous archwire inserted into multiple orthodontic brackets is poorly understood. The purpose of this research was to apply the birth-death technique to simulate the insertion of an orthodontic wire and the consequent transfer of forces to the dentition in an anatomically accurate model. **Methods:** A digital model containing the maxillary dentition, periodontal ligament, and surrounding bone was constructed from computerized tomography data. Virtual brackets were placed on 4 teeth (central and lateral incisors, canine, and first premolar), and a steel archwire (0.019 × 0.025 in) with a 0.5-mm step bend to intrude the lateral incisor was virtually inserted into the bracket slots. Forces applied to the dentition and surrounding structures were simulated by using the birth-death technique. **Results:** The goal of simulating a complete bracket-wire system on accurate anatomy including multiple teeth was achieved. Orthodontic forces delivered by the wire-bracket interaction were 19.1 N on the central incisor, 21.9 N on the lateral incisor, and 19.9 N on the canine. Loading the model with equivalent point forces showed a different stress distribution in the periodontal ligament. **Conclusions:** The birth-death technique proved to be a useful biomechanical simulation method for placement of a continuous archwire in orthodontic brackets. The ability to view the stress distribution with proper anatomy and appliances advances our understanding of orthodontic biomechanics. (Am J Orthod Dentofacial Orthop 2013;143:281-90)

Despite the use of fixed appliances for over 100 years, our understanding of orthodontic biomechanics is still limited. Prediction of the forces and moments transferred to the dentition by a continuous wire is generally limited to a 2-dimensional force diagram containing only 2 or 3 teeth without accurate anatomy (eg, periodontal ligament [PDL] and alveolar bone).[1] Free body diagrams assuming rigid body motion of archwires and teeth have been used to estimate forces and moments. Beyond this point, the system frequently becomes indeterminate.[2]

New advances in 3-dimensional (3D) technology, such as computer-aided design and computerized tomography imaging, allow for a more accurate description of dental anatomy. Although the associated force transfer through the dentition during orthodontic treatment frequently is statically indeterminate, these systems can be solved by incorporating the principles of solid mechanics. However, current finite element analysis that could predict applied forces with a continuous archwire is rarely combined with 3D multiple tooth systems.

Several appliance systems (Invisalign: Align Technology, Santa Clara, Calif; Insignia: Ormco Co, Orange, Calif; Incognito: 3M Unitek, St. Paul, Minn; and SureSmile: Orametrix Inc, Richardson, Tex) based on computer-aided design and computer-aided manufacturing already have computer models for appliance design purposes, but they do not focus on portraying forces produced by the appliances or transferring these forces to accurate anatomy. Huiskes and Chao,[3] Lin et al,[4] and Cattaneo et al[5] have reported that accurate anatomy of the patient-based models is important. Field et al[6] concluded that multiple teeth should be included in a model to allow the transfer of forces through both contact areas and interproximal tissues. Proper morphology of appliances and ligation methods are

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[a] Postgraduate student, Department of Orthodontics, School of Dentistry, University of North Carolina, Chapel Hill, NC.
[b] Teaching fellow, Department of Orthodontics, School of Dentistry, University of North Carolina, Chapel Hill, NC.
[c] Associate professor, Department of Orthodontics, School of Dentistry, University of North Carolina, Chapel Hill, NC.
[d] Private practice, Omaha, Neb.
[e] Professor, Department of Orthodontics, School of Dentistry, University of North Carolina, Chapel Hill, NC; adjunct professor, Department of Materials Sciences and Engineering, Engineering School, North Carolina State University, Raleigh, NC.

Drs. Canales and Larson have an equal contribution to this work.

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Reprint requests to: Ching-Chang Ko, Department of Orthodontics, University of North Carolina, CB#7450, Chapel Hill, NC 27599-7450; e-mail, koc@dentistry.unc.edu.

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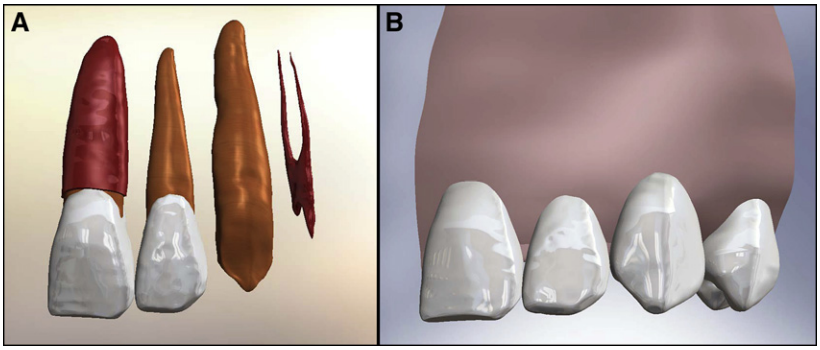

Fig 1. Final anatomy used for simulation: **A,** the different layers of tooth structure in the model, including the complete left central incisor with the PDL, the complete left lateral incisor without the PDL, the left canine dentin, and the left first premolar pulp; **B,** complete model with all 4 teeth and supporting tissues, including cortical bone, trabecular bone, and lamina dura.

important as well to accurately predict responses in 3 planes of space.

Current simplified finite element modeling often applies a point force to simulate orthodontic appliances rather than including the appliances.[5-10] Kim et al,[11] Mo et al,[12] and Sung et al[13] included brackets and wires to study the retraction of the anterior segment; however, the teeth were bonded together at the contact points and between the wire and the braces rather than individually ligated to the archwire with contact interfaces. These point-force models did not recreate the unloading force that is expressed by orthodontic wires in the clinic.

In this study, we propose a new model that includes the proper morphology of the teeth, surrounding tissues, and appliances. It is unique in that it examines the behavior of an activated continuous archwire that includes individual ligation of each tooth in the model rather than an archwire that, for modeling purposes, is assumed to be indistinguishable from "brackets" (bonded interfaces) as in the point-force models. Much as fence posts secure the fence rails or railroad ties secure the railroad tracks, this new model includes the brackets to secure the continuous archwire to allow the teeth to slide and thus has been named the rail-and-brace (ie, fence posts or railroad ties) model. Generating this more inclusive model will allow more accurate prediction and quantification of forces from appliances used in modern practice.

To simulate the effect of including the bracing structures (fence posts or railroad ties) in the model, a computer technique called the "birth-death" technique (ANSYS, Canonsburg, Pa) was used. It will be discussed in detail below but, in brief, refers to a 2-step technique of initially ignoring 1 aspect of the model (by "killing" it with the "death" step) and then adding it back in (the "birth" step). This is necessary because, in the virtual or computational world, unrealistic or impossible actions can occur, such as the step bend passing through a bracket as if the bracket were not present. In this model, the bracket is initially "killed" in the "death" step when the archwire is activated. Forces that result from the bracket's engaging the step bend of the archwire are ignored until the bracket is added back into the model in the "birth" step.

The purpose of this study was to simulate the activation of a step bend in an orthodontic archwire by using a virtual 3D finite element model with complete orthodontic appliances on 4 teeth. The specific aims were to place complete standard bracket and wire systems on accurate anatomy including multiple teeth, and to transfer and analyze the forces throughout the dentition and surrounding structures by clinically realistic wire bends with the birth-death technique.

MATERIAL AND METHODS

A virtual maxilla, including cortical bone, trabecular bone, and sinuses, was constructed by sequentially stacking cone-beam computerized tomography slices of a dentate maxilla. In a similar manner, the dentition was built by using microcomputed tomography templates by identifying the pulp, dentin, and enamel on sequential slices. The PDL (0.25 mm thick) and the lamina dura (0.5 mm thick) were added around each tooth with software (Solidworks, Concord, Mass). The models were merged by Boolean operations. Figure 1 shows the details of the model.

Three-dimensional computer-aided design models of 0.022-in slot, standard (0° tip, torque, and rotation) labial brackets on the maxillary right central and lateral incisors, canine, and first premolar were constructed and placed so that the gingival base of each slot was on the same reference plane. The bases of the bracket pad were adjusted to ensure a smooth interface with the facial surface of the tooth. No cement layer was included because it was

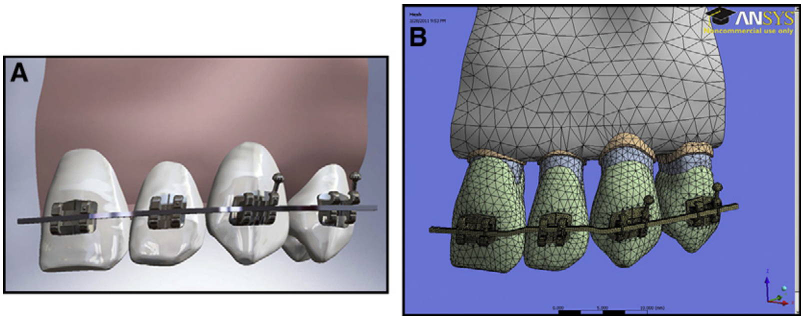

Fig 2. A, Final 3D Solidworks model of 0.022-in slot, standard labial brackets, and passive 0.019 × 0.025-in archwire placed on the 4-tooth maxillary model; **B,** model meshed in ANSYS.

deemed to have little effect unless a stress sufficient to debond the bracket was achieved.

The lingual and occlusal walls of the brackets were used as reference planes to create a passive 0.019 × 0.025-in stainless steel archwire in Solidworks (Fig 2, *A*). By using the passive wire as a starting point, a 0.5-mm intrusion bend was formed at the lateral incisor position. Bends were placed over a 1.0-mm section of wire in the adjacent interproximal areas. Two 0.010-in cylinders were added at either end of the slot opening to simulate passive stainless steel ligature wires that hold the archwire inside the bracket slot. The ligatures were placed to touch but not to overlap the archwire, and were combined with each bracket to form 1 body. The complete computer-aided design model was established with the Solidworks software, and the solid body information was saved in Initial Graphics Exchange Specification (IGES) format.

The IGES geometry files were imported into ANSYS version 13.0 Workbench. By using ANSYS Design Modeler, the individual solid bodies (bracket, ligature wire, enamel, dentin, pulp, PDL, lamina dura, and trabecular and cortical bones) were merged into a multibody part, allowing for conformal meshing of the model. Conformal meshing creates shared nodes at the interfaces, providing accurate modeling for bonded heterogeneous biologic structures. The archwire was not merged into this multibody part, allowing for frictionless contacts areas with nonconformal meshing and wire movement relative to the brackets. In addition, the interproximal contacts were also considered frictionless, so that the teeth were allowed to move relative to each other. All other interfaces were rigidly bonded by using the penalty method. Proper material properties were assigned for enamel, dentin, pulp, PDL, cortical bone, trabecular bone, and stainless steel (Table I[14-19]), with the assumption that all materials were isotropic and linearly elastic. The final model was meshed by using the tetrahedral 10-node element, except for the swept hexahedral 8-node element in the archwire, and consisted of 238758 nodes and 147747 elements (Fig 2, *B*). The model was fixed at the ends of the archwire and at the cortical and trabecular bone at the midline and distally to the first premolar.

Table I. Material properties

	Poisson's ratio	*Young's modulus (Pa)*
Enamel	0.41	8.00E + 10
Dentin	0.31	1.80E + 10
Pulp	0.3	1.75E + 08
PDL	0.3	1.75E + 09
Cortical bone	0.31	1.37E + 10
Trabecular bone	0.3	1.37E + 09
Stainless steel	0.3	2.00E + 11

The static equilibrium equations were solved under the large displacement assumption. Deformation of the system was evaluated to confirm the validity of the solution.

To simulate the insertion of an active archwire, a 2-step computer technique called "birth and death" was used. Since the archwire with the 0.5-mm step bend initially overlaps the lateral incisor bracket, the first step involves displacing the wire into the bracket slot. This requires intentionally ignoring the interaction between elements in the contact area involving the wire and bracket, known as a "kill" or "death" step. Displacement of the wire into the slot removes the overlap with the bracket and loads stored energy in the wire and adjacent teeth (Fig 3). With this solution, the deactivated elements of the contact area were reactivated, known as an "alive" or "birth" step. Then, in this second step, the displacement of the wire was relaxed, loading the lateral incisor (Fig 4). This "birth and death" function allows elements to change status in contact areas at a later step in the simulation; this is required for insertion of active archwires.

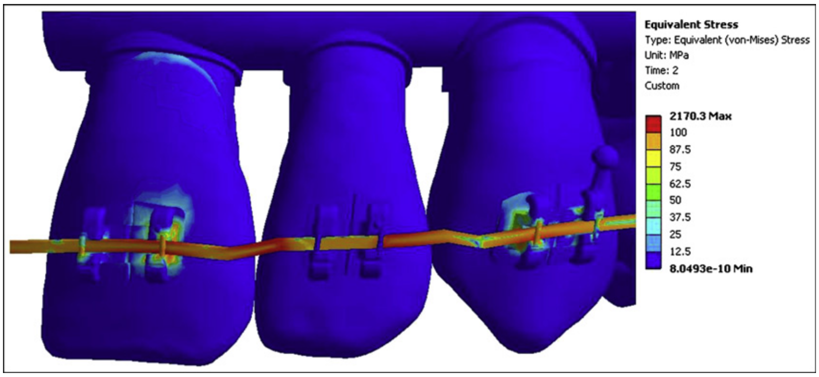

Fig 3. Equivalent (von Mises) stress (MPa) at the end of the first step with the contact deactivated and the wire displaced into the bracket slot. High stresses are seen in the central incisor and canine brackets.

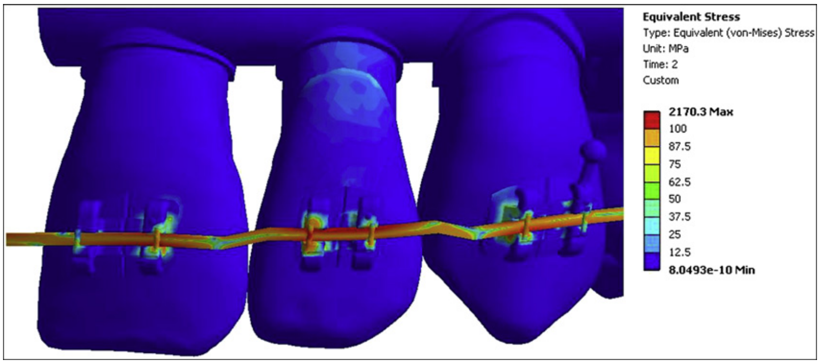

Fig 4. Final equivalent (von Mises) stress (MPa) after reactivation of the contact elements and allowing the wire to load the lateral incisor.

Table II. Bracket-wire reaction forces (N) during the birth-death technique

	Central incisor	*Lateral incisor*	*Canine*
Step 1	40.1	0	18.8
Step 2	19.1	21.9	19.9

Static structural solutions were converged at 2 steps (element kill and element birth), and the results were generated for reaction forces, maximum and minimum principal elastic strains, equivalent elastic strain, maximum and minimum principal stresses, equivalent (von Mises) stress, and total deformation. Convergence of the solution was checked by using the Newton-Raphson method.[20]

To compare the birth-death technique with previously published point-force models, a second model was generated with identical anatomy. In this model, the archwire was removed, and point forces were applied by using the reaction forces calculated from the birth-death model. Stress distribution through the dentition and the surrounding tissue was compared with the birth-death model.

RESULTS

The solution converged for both steps in the birth-death model. Before the contact area was activated in the lateral incisor bracket, displacement of the wire placed stress on the canine and central incisor brackets (Fig 3). The lateral incisor was not loaded in this step because the elements were deactivated. Additionally, more stress was seen in the central incisor bracket than in the canine and premolar brackets, since the activation force was primarily placed on 1 bracket instead of evenly distributing to 2 brackets. Once the contact was reactivated, the stress increased in the lateral incisor bracket and decreased in the adjacent brackets (Fig 4). The final solution showed higher force levels (Table II) in the lateral incisor bracket than in the central incisor and canine brackets.

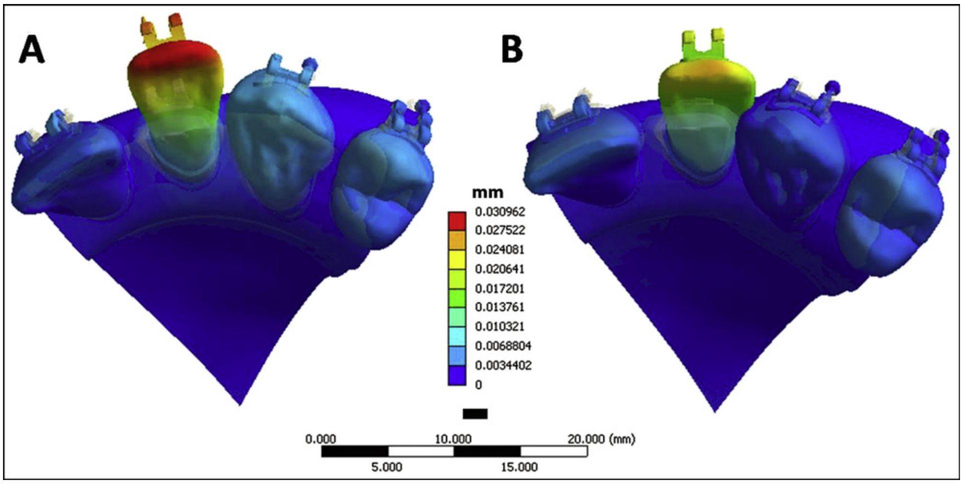

Fig 5. Converged solutions for displacement (mm) in: **A,** the birth-death model and **B,** the point-force model. The overall displacement is exaggerated 200 times to better visualize the side effects.

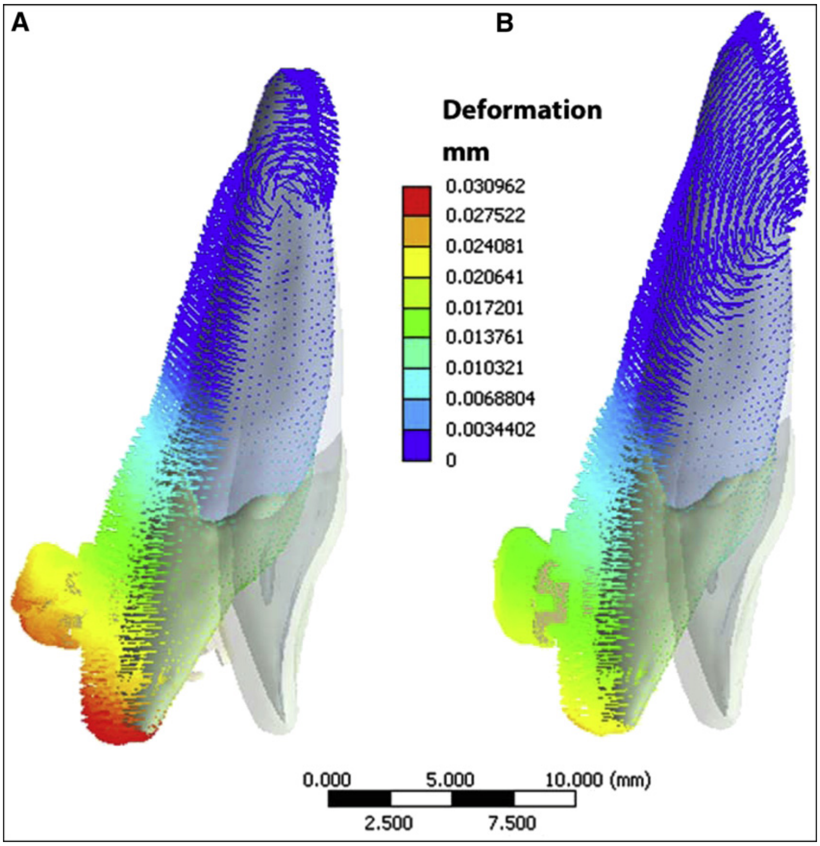

Fig 6. Center of rotation for the lateral incisor in: **A,** the birth-death model and **B,** the point-force model. The total displacement (mm) is shown graphically, emphasized 200 times.

Overall tooth displacement differed from the expected results with the free body diagrams—intrusion and labial tipping of the lateral incisor, and extrusion and lingual tipping of the central incisors and the canine. The lateral incisor was intruded and experienced labial tipping since the force was facial to the center of

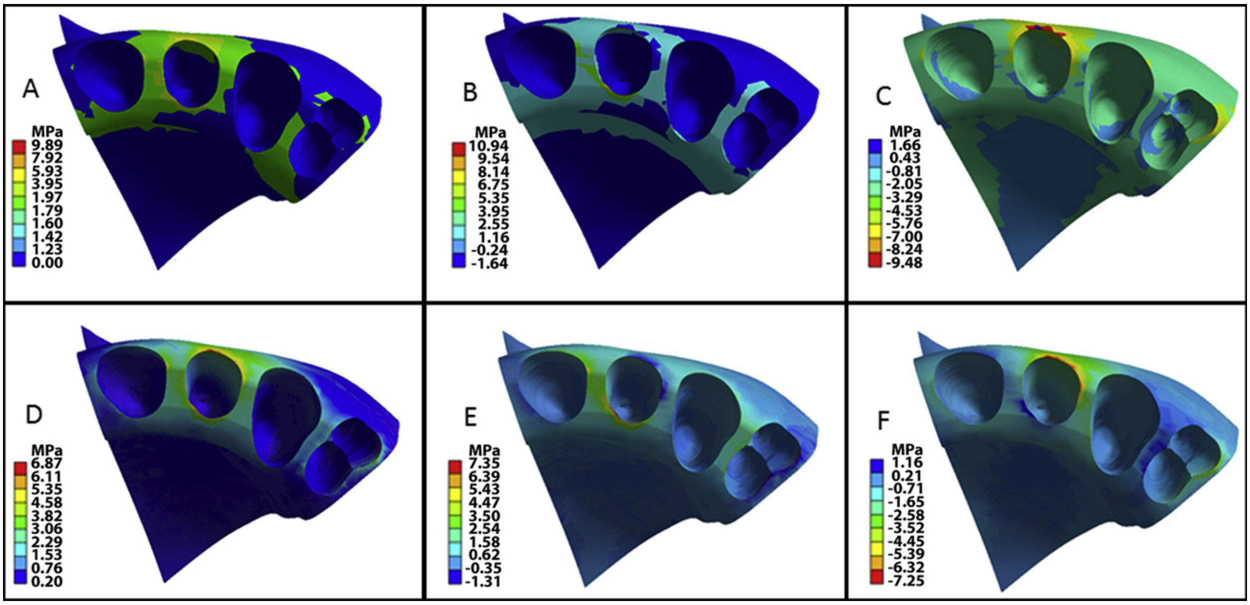

Fig 7. Stress (MPa) distribution in alveolar bone and tooth sockets; teeth, brackets, wire, and PDL are hidden. The birth-death model (**A**-**C**) showed higher stress levels than did the point-force model (**D**-**F**). **A** and **D,** Equivalent (von Mises) stresses; **B** and **E,** tensile stresses; **C** and **F,** compressive stresses.

resistance (Fig 5, *A*). Unexpectedly, a small distal displacement was also seen. The central incisor and the canine were slightly extruded and tipped 0.00085 and 0.000023 mm facially, respectively, although a slight lingual crown tipping would be expected, since the force application was located facially to the center of resistance.

The results from the point-force simulation showed significant differences from the birth-death model. First, the overall displacement was 28% less in the point-force simulation (Fig 5). The central incisor, lateral incisor, and canine all displaced further facially in the birth-death model, possibly from the wire lengthening because of flattening of the activated bend. In the point-force model, the lateral incisor was also slightly to the distal aspect, possibly because the bracket was slightly distal to the center of resistance. However, the centers of rotation of the lateral incisor were different between the 2 models (Fig 6). When the birth-death technique was used to accurately model the bracket-wire interface, the center of rotation moved apically compared with the point-force model.

The stress distribution in the surrounding bone was different between the 2 models (Fig 7). The birth-death model showed compressive stress (1-2 MPa) uniformly distributed over the buccal cortical bone across all 4 teeth, but the point-force model showed tensile stress in the buccal bone of the central incisor and the canine. Both models had the compressive stress concentrated on the buccal cortex of the lateral incisor, with 19% higher levels for the birth-death model. The tensile stresses concentrated in the lingual alveolar crest of the lateral incisor were 40% greater in the birth-death model than in the point-force model. As a result, the von Mises stresses were 47% greater for the birth-death model.

DISCUSSION

In the clinic, a step bend provides intrusive force and hence intrusion to at least 1 tooth. Since the archwire is active, force is required to insert the archwire into the bracket slot. This creates stored elastic energy that is then released during unloading of the wire and tooth movement. Simulating this insertion and unloading by using computer modeling can be challenging. A unique method that can accomplish these mechanics, the birth-death technique, has not been previously used in orthodontic biomechanical studies.

The birth-death method combines placement of accurate computer-aided designed brackets on anatomically correct computer models and manipulation of contact conditions; this allows virtual placement of active archwires into bracket slots. Simulation of active archwires was not previously possible. This model allows for clinically relevant examination of complications associated with intrusive bends. For example, unexpected tooth displacements were found, including slight distal movements of the lateral incisor and facial tipping of the central incisor and the canine. Both compression

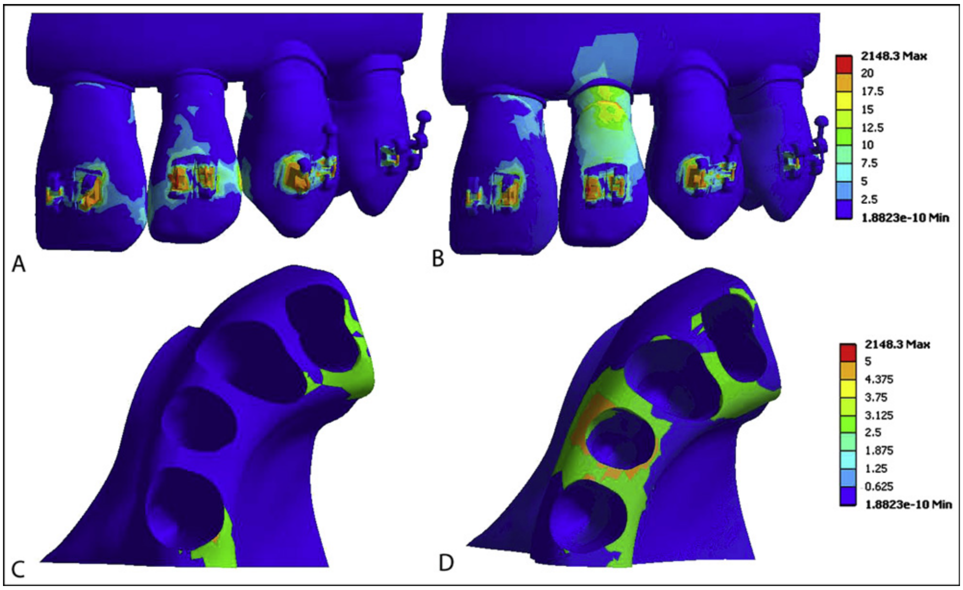

Fig 8. Equivalent (von Mises) stress (MPa) between the central and lateral incisors when the brackets are loaded. The orthodontic wire is hidden. **A** and **C,** All adjacent teeth were bonded at the contact areas; **B** and **D**: the adjacent teeth were simulated as frictionless contact. **A** and **B** show von Mises stresses (MPa) on the teeth; the stress fringe values are shown on the right. **C** and **D** show von Mises stresses (MPa) in alveolar bone; the teeth are hidden.

and tension stresses were underestimated by the conventional point-force model. Our model has advanced finite element simulation of orthodontic biomechanics, profoundly increasing the complexity of computing contacts between teeth and within orthodontic appliances.

Often finite element models do not include brackets or archwires. Without appliances, forces and moments are often applied at a point near an ideal bracket position to simulate the appliances.[5-10] Tanne et al[10] examined the behavior of the PDL with a force applied at 1 point on the buccal surface. Cattaneo et al[5,8,9] constructed detailed finite element models and used point forces without including bracket-wire systems. None of these previous models could predict unwanted tooth displacements with associated wire bending. Although these studies help us to understand how the PDL is loaded, ultimately, bracket-wire appliances must be included to understand the clinically relevant interactions between multiple teeth and appliances.[3] The birth-death technique is the first reported method of achieving this goal.

Finite element models require a balance of creating proper anatomic accuracy without requiring unrealistic computational central processing unit time to obtain a solution. Previous studies have used microcomputed tomography scans with a voxel dimension of 37 μm and applied intensity segmentation to form a 3D model with Mimics software and found that level of detail to be important.[5,8,9] In this study, we generated a model from high-quality microcomputed tomography scans with a similar voxel size but further refined the model to allow manipulation of the contact conditions.

In this study, lower stress was seen at the contact points between the teeth compared with the model with bonded teeth (Fig 8). In addition, higher levels of stress were transferred through the alveolar bone between adjacent teeth; this could potentially stimulate bone remodeling for orthodontic tooth movement. The bonded teeth model predicted stress concentrations at contacts of the adjacent teeth consistent with the study of Field et al,[6] who constructed a 3D computer-aided design model of cortical bone, alveolar bone, enamel, and dentin based on computed tomography images of an adult mandible. Field et al analyzed both 1 tooth and a 3-tooth segment after the addition of brackets and wires. Their results had limited crown displacements in the multiple-tooth model and more displacement in the 1-tooth model, showing that "adjacent teeth significantly alter stress distribution and their magnitudes."[6] Field et al concluded that the inclusion of multiple teeth in a finite element model transferred a force at the interproximal contacts of adjacent teeth but also through the PDL and alveolar bone. In summary, the bonded tooth model underestimated the stress trajectory passing

Fig 9. Clinically recorded archwire deformation (facial view photo) matches the computer (birth-death model) predicted wire deformation (*colored mapped wire with blue brackets*). Each *white-scale interval* (right) stands for 1 mm. The *color fringe* on the left represents the deformation (mm) map by the computer prediction.

through the alveolar bone and could not accurately predict bone remodeling.

Both of these findings indicate that multiple teeth and inclusion of the archwire with the rail-and-brace structure are important when modeling orthodontic forces. Our findings were also supported by those of Cattaneo et al.[5] However, authors of previous studies rigidly bonded contacts together, whereas there was a frictionless contact area in our study. This led to a stress concentration at the alveolar crest as the teeth were loaded.

In finite element studies that do include appliances, contact points between teeth are often rigidly fixed; this can affect the results depending on the tooth movements being studied. Examples include several recent studies that used archwires and brackets in the finite element models to examine en-masse retraction of anterior teeth into a first premolar extraction site. Kim et al[11] studied the length and position of the power arm needed for parallel translation of anterior teeth. Sung et al[13] examined mini-implant position, anterior retraction hook position, and compensating curve. Mo et al[12] studied the length of the retraction arm and the degree of the gable bend when using a C-implant as the exclusive source of anchorage in retraction of the anterior segment. In each of these studies, the focus was on a segment of teeth bound at the contact points. Our model looks closely at the interaction between the archwire and brackets, whereas individual teeth are not bound together at the contact points. Allowing both the teeth and the archwire the freedom to move independently is important in simulating the tooth response to the appliances.

The force magnitudes (19.1-21.9 N) that resulted from the 0.5-mm step bend of a 0.019 × 0.025-in archwire appear too heavy to intrude the incisor according to the light-force concept.[21] Nevertheless, the current prediction is within the reasonable range based on the simple beam theory (3-point bending). According to the theory, $F = 48EIv/L^3$, where F is the point force at the middle of the beam, E is Young's modulus of stainless steel, I is the moment of inertia, v is the vertical displacement (step-bend depth), L is the beam span, and the *48* is the support constant.[22] The two 3-point bend analogs are the 7-mm step-bend span and the 15-mm interbracket distance between the central incisor and the canine; they predict forces of 79 and 8 N, respectively. This finite element model, predicting the forces of 19.1 to 21.9 N, is within the theoretic values. The simple beam theory partially provides validity for this model with frictionless contacts. On the contrary, a fixed support beam (binding between the wire and the brackets), with a support constant of 192, would predict a higher force level compared with our model. Perhaps ligation effects and friction at the bracket-wire interface would increase the constant in the bending equation; this would increase the loads.

Clinically recorded archwire deformation perhaps could serve as the best validation for this model. Although we did not have quantitative deformation data to validate the model, a clinical photo was taken to compare with the shape of wire deformation of the prediction (Fig 9). Qualitatively, the shape of the intrusive bend was similar between the computer prediction and the clinical photo from a superimposition of the facial view. The matched archwire deformation partially supported the birth-death simulation, which warrants future clinical trials for the study of clinical biomechanics and computational biomechanics with 3D quantitative images.

The predicted compressive stress values on the lamina dura (socket) of the lateral incisor ranged from 500 to 2000 KPa with the associated high reactant force (20 N). Assuming that one can use a smaller archwire to reduce the intrusive force to 0.2N (20 g), the stress should proportionally decrease to 5 to 20 KPa that is consistent with the stress value (4-13 KPa) for proper tooth movement previously reported by Iwasaki et al.[23] The peak von Mises stresses that were frequently used to predict failure of engineering materials were 9.28 MPa for the birth-death model and 6.28 MPa for the point-force model located at the facial alveolar crest of the lateral incisor. Similarly, the magnitudes of von Mises stress could drop to 0.09 and 0.06 MPa as the less rigid archwire was used. It is unlikely that these stresses would cause microdamage to bone because the yield strength of cortical bone is approximately 100 to 170 MPa.[24]

Further manipulation of this model can advance our understanding of the orthodontic biomechanics used with various appliances. In addition, this study demonstrates the technological capability of modeling the anatomy of the alveolar dental complex and then viewing, in detail, the forces applied from and throughout the appliances. If coupled with the computer-aided design and manufacturing procedures used to make custom appliances (Invisalign, Insignia, Incognito, and SureSmile), perhaps a custom appliance can be designed to apply specified force levels to the teeth and ideal pressure to the PDL.

The limitations of this study include the frictionless assumption between the archwire and the bracket. Adding friction when modeling multiple teeth increases the complexity of the computations; this requires future investigations. The absence of friction might have underestimated the distal and mesial forces on the intruded and adjacent teeth. The frictionless assumption partially eased the wire expansion effect that might be caused by the activated step bend with the fixed boundary condition at both ends of the archwire. Nevertheless, our model already predicted distal displacement of the lateral incisor that could not be estimated by the conventional point-force model. Such information is clinically important. Another limitation of our model is the linear elastic assumption of the PDL's property. Brosh et al[25] found that 82% of the elastic behavior of the PDL was regained in the first minute, and 6% of the viscous response occurred 30 minutes later. Our assumption allowed investigation of initial tooth loading, which accounts for the first minute of tooth movement with the PDL effect that cannot be simulated with the most recent laboratory simulator.[26] Recent reports indicate that mathematical modeling with rigorous assumptions can provide more accurate results approximating clinical situations than do laboratory tests.[27,28] Future expansion to a full arch and full dentition model would advance orthodontic biomechanics.

CONCLUSIONS

The birth-death technique was used to create a new method to simulate the clinical effects of inserting an archwire in brackets to allow force to be transferred to the surrounding dental structures. We named this method the rail-and-brace model. It might be more valid than the point-force model and should be considered for future investigations of computer simulations of orthodontic force applications.

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